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Cavity-Assisted Interaction of Superconducting Charge Qubits: The Role of ac Magnetic Flux

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Abstract We propose, in analogy with trapped ions, scalable quantum computation schemes with superconducting charge qubits couple to a micro-wave cavity mode. Single-qubit addressing can be achieved and selective qubit-cavity coupling can be effectively controlled by the external magnetic flux, thus gate operations can be selectively performed. During the implementation of a certain (virtual) excitation operation all the qubits and cavity parameters can be chosen to be fixed, the only parameter needs to be tunable is the external magnetic flux. This is a more efficient way of controlling the system dynamics as it is much easier for experimental realization.

Keywords Quantum computation · Superconducting charge qubits · Ac magnetic flux

Physical implementation of quantum computers has attracted much attention in the last decade as they are generally believed to be capable of solving diverse classes of hard problems. Systems suitable for hardware implementations of quantum computers should possess relatively long coherent time, easy manipulation and good scalability. Cavity quantum electrodynamics (QED) system is favored because of its demonstrated advantage when subjected to coherent manipulations [1–3], but it is difficult to realize the strong-coupling using atomic qubit. Trapped ions approach of quantum computation [4–7] is further ahead of other qubits along the quantum computing Roadmap, but the scalability of trapped ions is not good. With highly developed fabrication techniques, the superconducting quantum interference device (SQUID) has shown its competence in implementing the qubit for scalable quantum computation [8–10]. The idea of placing the superconducting qubits in a cavity has been illustrated to have several practical advantages including strong coupling strength, immunity to noise, and suppressed spontaneous emission.

Quantum computation with cavity-assisted interaction with superconducting qubits [11–29] is an analogy to cavity QED and trapped ions approaches. Zhu et al. [24] can be understood as analogy of the original trapped ions quantum computation scheme presented by

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Cirac and Zoller (CZ) [4]. The scheme realized the excitations by tuning the qubit parameter and the cavity-qubit interaction is of the *always on* nature, that is, there lacking of effective switch method. It was shown [28, 29] that by introducing the ac external magnetic flux the effective switch on/off cavity-qubit interaction can be selective achieved, thus may result in universal quantum computation and state transfer among selective qubits. In this spirit, Zhu et al. [28] presented successful analogy of quantum computation with thermal trapped ions presented by Sørensen and Mølmer (SM) [5, 6] with a new designed 4 junction qubit. This effective switch method has also be investigated using the 1D transmission line resonator cavity [23] as well as superconducting quantum circuit [30–32] where a *LC* circuit play the role of the "vibrating" mode in trapped ions quantum computation schemes. It is also noted that another interesting switch mechanism is presented in [33], where a frequency-matching technique is adopted. In addition, [32, 33] are also the superconducting quantum circuit analogy of the CZ scheme.

Here, we propose the fully analogy of the CZ and SM trapped ions quantum computation schemes with SQUID charge qubits couple to a micro-wave cavity mode. By using the ac external magnetic flux as the effective switch method, single-qubit addressing can be achieved and selective qubit-cavity coupling can be effectively controlled, thus gate operations can be selectively performed. During the implementation of the operations all the qubits and cavity parameters can be chosen to be fixed, the *only* parameter needs to be tunable is the external magnetic flux. This is a more efficient way of controlling the system dynamics as it is much easier for experimental realization in principle.

A single superconducting qubit considered here consists of a small superconducting box with excess Cooper-pair charges, formed by an symmetric SQUID with the capacitance C_J and Josephson coupling energy E_J, pierced by an external magnetic flux Φ. A control gate voltage V_g is connected to the system via a gate capacitor C_g. The Hamiltonian of the system is [8, 9]

$$H = E_c(n-\bar{n})^2 - E_J\cos\varphi_1 - E_J\cos\varphi_2, \tag{1}$$

where n is the number operator of (excess) Cooper-pair charges on the box, $E_c = 2e^2/(C_g + 2C_J)$ is the charging energy, $\bar{n} = C_gV_g/(2e)$ is the induced charge controlled by the gate voltage V_g, and φ_m $(m = 1, 2)$ is the gauge-invariant phase difference between the two sides of the mth junction. We here focus on the charge regime, where the quasiparticle tunneling or excitation can be effectively suppressed. In this case, a convenient basis is formed by the charge states, parameterized by the number of Cooper pairs n on the box with its conjugate φ; they satisfy the standard commutation relation: $[\varphi, n] = i$. At temperatures much lower than the charging energy and restricting the gate charge to the range of $\bar{n} \in [0, 1]$, only a pair of adjacent charge states $\{|0\rangle, |1\rangle\}$ on the island are relevant. The Hamiltonian (1) is then reduced to

$$H_s = -E_{ce}\sigma^z - E_\Phi\sigma^x, \tag{2}$$

where $E_{ce} = E_c(1-2\bar{n})/2$, $E_\Phi = E_J\cos\phi$, $\phi = \pi\Phi/\phi_0$ with ϕ_0 being the flux quanta, σ^x and σ^z are the Pauli matrices.

When a superconducting qubit is placed in a cavity the gauge-invariant phase difference becomes $\varphi'_m = \varphi_m - \frac{2\pi}{\phi_0}\int_{l_m}\mathbf{A}_m\cdot d\mathbf{l}_m$, where $\mathbf{A}_m$ is the vector potential in the same gauge of φ_m. $\mathbf{A}_m$ may be divided into two parts $\mathbf{A}'_m + \mathbf{A}^\phi_m$, where the first and second terms arise from the electromagnetic field of the cavity normal modes and the external magnetic flux, respectively. For simplicity, we here assume that the cavity has only a single mode to play a role. Therefore, we have $\frac{2\pi}{\phi_0}\int_{l_m}\mathbf{A}_m\cdot d\mathbf{l}_m = \frac{2\pi}{\phi_0}\int_{l_m}\mathbf{A}^\phi_m\cdot d\mathbf{l}_m + 2g(a+a^\dagger)$, where ϕ_0 is the flux quantum, $2g$ is the coupling constant between the junctions and the cavity, a and $a^\dagger$ are the annihilation and creation operators for the cavity mode, and the closed path integral of the $\mathbf{A}^{\phi}$, with l_m the integral path and can be chosen as the contour of the qubit, gives rise to the magnetic flux Φ.

Firstly, we focus on the dc magnetic flux case, which can realize the so-called *carrier process* in trapped ions quantum computation. The Hamiltonian (1) reads

$$H_c^1 = -E_{ce}\sigma_z - E_J \cos\left[\phi + g\left(a + a^{\dagger}\right)\right]\sigma_x. \tag{3}$$

Generally speaking, $g \ll \phi$, then the Hamiltonian (3) approximately reads

$$H_c^2 = -E_{ce}\sigma_z - E_{\Phi}\sigma_x + g \sin\phi\left(a + a^{\dagger}\right)\sigma_x. \tag{4}$$

In the rotating frame with respect to $H_0 = -E_{ce}\sigma_z - E_{\Phi}\sigma_x + \hbar\omega_c(a^{\dagger}a + \frac{1}{2})$, the interaction term in the Hamiltonian (4) osculating with the frequency of $\omega_c - \omega_q$ with $\hbar\omega_q = 2E_{ce} = E_c(1-2\bar{n})$. As $\bar{n} \in [0, 1]$, then $\omega_q \in [-E_c/\hbar, E_c/\hbar]$, which is depends on the induced charge $\bar{n}$ and thus can be controlled. In our present scheme, we choose ω_q as a small quantity, if not zero, as we need to maintain the large detuning limits ($\omega_q \ll \omega_c$). In addition, $\omega_q = 0$ corresponds to the degeneracy point ($\bar{n} = 1/2$) where the qubit possess long coherent time and minimal charge noise. Given the facts that g is relatively very small and the frequency difference $\omega_c - \omega_q$ was chosen to satisfy the large detune limits, then the interaction of qubit and the cavity can be safely neglected. In addition, from (4), the interaction can be exactly switched off by choosing $\Phi = 2k\phi_0$ with k an integer. In other words, the qubit and the cavity evolve independently in this case, the external flux is only used to separately address the qubit rotations and the evolution of the qubit are governed by the Hamiltonian in (2). This is similar to the so-called *carrier process* in the trapped ions approach of quantum computation, universal single-qubit rotations are possible from (2) using standard method [8, 9].

Then we focus on the ac magnetic flux case, which can realize both the *red and blue excitations* in trapped ions quantum computation. If N qubits are located within a single-mode cavity. To a good approximation, the whole system can be considered as N two-level systems coupled to a quantum harmonic oscillator [18]. Setting $\phi = \omega t$, the system considered here can then be described by the Hamiltonian $H = H_0 + H_{int}$ with

$$H_0 = \hbar\omega_c\left(a^{\dagger}a + \frac{1}{2}\right) + \frac{\hbar}{2}\omega_q\sum_{j=1}^{N}\sigma_j^z, \tag{5a}$$

$$H_{int} = -\frac{E_J}{2}\sum_{j=1}^{N}\left(\sigma_j^+ e^{-\mathrm{i}[g(a+a^{\dagger})+\omega t]} + \mathrm{H.c.}\right) \tag{5b}$$

where $\sigma^{\pm} = \frac{1}{2}(\sigma^x \pm \mathrm{i}\sigma^y)$. For simplicity, we have also assumed the same E_c and E_J for all qubits. Expanding the Hamiltonian (5b) to the first order of g in the Lamb-Dicke limit and under the rotating-wave approximation as well as in the interaction picture with respect to $U_0 = \exp(-iH_0t)$, the Hamiltonian is reduced to

$$H_e = \frac{\mathrm{i}gE_J}{2}\sum_{j=1}^{N}(\sigma_j^+ e^{\mathrm{i}\omega_q t} + \sigma_j^- e^{-\mathrm{i}\omega_q t})\left(ae^{-\mathrm{i}\omega_c t} + a^{\dagger}e^{\mathrm{i}\omega_c t}\right)\left(e^{-\mathrm{i}\omega t} - e^{\mathrm{i}\omega t}\right). \tag{6}$$

If the frequencies satisfy the condition $\omega = \omega_q - \omega_c$, then the ac external magnetic flux ϕ in the qubits assist the qubit resonantly couple to the cavity mode. This is the *red sideband* *excitation* governed by the Hamiltonian

$$H_r = \frac{\mathrm{i}gE_J}{2}\sum_{j=1}^{N}\left(a\sigma_j^+ - a^\dagger\sigma_j^-\right). \tag{7}$$

When the frequencies satisfy the condition $\omega = \omega_q + \omega_c$, it is the so-called *blue sideband excitation* governed by the Hamiltonian

$$H_b = \frac{\mathrm{i}gE_J}{2}\sum_{j=1}^{N}\left(a^\dagger\sigma_j^+ - a\sigma_j^-\right). \tag{8}$$

It is clear that the qubit-cavity coupling, as well as decoupling, can be controlled by appropriately selecting the frequency ω to match the above frequency condition, not by changing the parameters of the qubit or the cavity. The external magnetic flux can be effectively controlled, and thus qubit-cavity interaction can be implement in selective qubit or qubits.

Choosing $\omega_q = 0$, both the red and blue excitation reduce to

$$H_{br} = \frac{gE_J}{2}\left(a + a^\dagger\right)\sum_{j=1}^{N}\sigma_j^x, \tag{9}$$

which is the *simultaneous realization* of Jaynes-Cummings and anti-Jaynes-Cummings interaction when $N = 1$. This type of interaction appearing naturally in trapped ions [7] but not in the context of cavity QED with atomic qubit. With this interaction, the entangled Schrödinger cat state can be generated to demonstrate the microscopic-mesoscopic entanglement [2].

Next, we investigate the red and blue detuning with $\omega_q = 0$ to implement SM type interaction presented in the context of quantum computation using thermal trapped ions [5, 6]. When the frequency of the external magnetic flux $\omega = \omega_c - \delta$, the Hamiltonian (6) is reduced to

$$H_r^d = \frac{\mathrm{i}gE_J}{2}\left(a^\dagger e^{\mathrm{i}\delta t} - ae^{-\mathrm{i}\delta t}\right)\sum_{j=1}^{N}\sigma_j^x, \tag{10}$$

where $\delta \ll \omega$. This is the so-called *red detuning interaction*, when the so-called periodical evolution ($\delta T = 2k\pi$) [28, 29] or large-detuning ($\delta \gg gE_J/2\hbar$) [25–27] condition is satisfied, the effective Hamiltonian for (12) is

$$H_x = \chi_x\left(\sum_{j=1}^{N}\sigma_j^x\right)^2, \tag{11}$$

where $\chi_x = g^2E_J^2/4\hbar\delta$. When in the case of *blue detuning* ($\omega = \omega_c + \delta$), the Hamiltonian (6) is reduced to

$$H_b^d = \frac{\mathrm{i}gE_J}{2}\left(a^\dagger e^{-\mathrm{i}\delta t} - ae^{\mathrm{i}\delta t}\right)\sum_{j=1}^{n}\sigma_j^x. \tag{12}$$

Under the large-detuning condition, the effective Hamiltonian for (10) is $-H_x$. We can see that both the *red and blue detuning* in the context of superconducting cavity QED reduce to the same type of coupling but with opposite eigenvalues.

Interestingly, this interaction is insensitive to the thermal state of the cavity mode. It is also notable that, it is straightforward to check that the evolution under Hamiltonian (11) is a unconventional geometric phase gate [28]. The paths of Hamiltonian (12) and (10) in xp phase space to be traversed are in the opposite direction, and thus the sign of the geometric phase is also opposite. This unconventional geometric phase shift still depends only on global geometric features and is robust against random operation errors [34]. More generally, it can be used to generate multipartite entangled GHZ state [28] and cluster states for one-way quantum computation [29, 35] as well as construct quantum error correcting code [28] with an unconventional geometric phase shift scenario.

Lastly, we investigate the red and blue detuning with $\omega_q \neq 0$ to implement the XY Hamiltonian with cavity mediated long-range coupling, which can also be realized in the context of atoms in cavity [3]. If the frequencies satisfy the condition $\omega = \omega_c + \delta$, then the Hamiltonian (6) is reduced to

$$H_r = -\frac{\mathrm{i}gE_J}{2}\sum_{j=1}^{N} a\left(\sigma_j^+ e^{\mathrm{i}\delta_+ t} + \sigma_j^- e^{\mathrm{i}\delta_- t}\right) + \mathrm{H.c.}, \tag{13}$$

where $\delta_\pm = \delta \pm \omega_q$. When the frequencies satisfy the condition $\omega = \omega_c - \delta$, then the Hamiltonian (6) is reduced to

$$H_b = -\frac{\mathrm{i}gE_J}{2}\sum_{j=1}^{N} a\left(\sigma_j^+ e^{-\mathrm{i}\delta_- t} + \sigma_j^- e^{-\mathrm{i}\delta_+ t}\right) + \mathrm{H.c.} \tag{14}$$

The effective Hamiltonian of Hamiltonian (13) and (14) will be in the XY type as in atomic cavity QED [3].

In implementing our scheme, several points need further clarification. (1) With the properly chosen parameters, both the Lamb-Dicke and strong-coupling limits can be satisfied simultaneously. Suppose that the quality factor of the superconducting cavity is $Q = 1 \times 10^6$; for the cavity with $\omega_c = 50$ GHz, the cavity decay time is $\tau = 3.2$ μs. With the vacuum Rabi frequency $\Omega = 8$ MHz and the decoherence time of the qubit $T = 0.5$ μs, the strong-coupling limit can be readily reached ($\Omega^2 \tau T \sim 10^2 \gg 1$). (2) Many coherent operations are possible. Typical operation time for large detuned interaction is on the order of ns (the time scale for the excited operations are much more faster), this ensures thousands of operations are possible before decoherence set in as the coherence time scales of the qubit and cavity mode are both on the order of μs. (3) The scheme is ready to scale up. The realistic length of the cavity is $l \sim 1$ cm. When the two SQUIDs are located each at one of the antinodes of the microwave, the distance of the two neighbor qubits is the wavelength of the microwave, which can be chosen as $\lambda \sim 1$ mm, then 10^3 qubit maybe constructed along the cavity direction. Furthermore, as the typical size of the SQUID loop is on the order of μm, that is, the distance of the two qubit is about 10^3 times of the qubit size, thus the mutual induction of the neighbor qubits can be safely neglected.

In summary, we propose the fully analogy of the CZ and SM trapped ions quantum computation schemes with superconducting charge qubits couple to a micro-wave cavity mode using the external magnetic flux as the effective switch method. The only parameter needs to be tunable is the frequency of the external magnetic flux. This simplified scheme with fixed cavity and qubit parameters is a more efficient way of controlling the system dynamics as it is much easier for experimental realization.

References

1. Raimond, J.M., Brune, M., Haroche, S.: Rev. Mod. Phys. **73**, 565 (2001)
2. Solano, E., Agarwal, G.S., Walther, H.: Phys. Rev. Lett. **90**, 027903 (2003)
3. Zheng, S.-B., Guo, G.-C.: Phys. Rev. Lett. **85**, 2392 (2000)
4. Cirac, J.I., Zoller, P.: Phys. Rev. Lett. **74**, 4091 (1995)
5. Sørensen, A., Mølmer, K.: Phys. Rev. Lett. **82**, 1971 (1999)
6. Sørensen, A., Mølmer, K.: Phys. Rev. A **62**, 022311 (2000)
7. Solano, E., de Matos Filho, R.L., Zagury, N.: Phys. Rev. Lett. **87**, 060402 (2001)
8. Makhlin, Y., Schön, G., Shnirman, A.: Nature (London) **386**, 305 (1999)
9. Makhlin, Y., Schön, G., Shnirman, A.: Rev. Mod. Phys. **73**, 357 (2001)
10. You, J.Q., Nori, F.: Phys. Today **58**(11), 42 (2005)
11. Kis, Z., Paspalakis, E.: Phys. Rev. B **69**, 024510 (2004)
12. Yang, C.-P., Chu, S.-I., Han, S.: Phys. Rev. Lett. **92**, 117902 (2004)
13. Yang, C.-P., Chu, S.-I., Han, S.: Phys. Rev. A **67**, 042311 (2003)
14. Yang, C.-P., Han, S.: Phys. Rev. A **72**, 032311 (2005)
15. Zhang, P., Wang, Z.D., Sun, J.D., Sun, C.P.: Phys. Rev. A **71**, 042301 (2005)
16. Song, K.-H., Zhou, Z.-W., Guo, G.-C.: Phys. Rev. A **71**, 052310 (2005)
17. Song, K.-H., Xiang, S.-H., Liu, Q., Lu, D.-H.: Phys. Rev. A **75**, 032347 (2007)
18. Al-Saidi, W.A., Stroud, D.: Phys. Rev. B **65**, 224512 (2002)
19. You, J.Q., Nori, F.: Phys. Rev. B **68**, 064509 (2003)
20. Wallraff, A., Schuster, D.I., Blais, A., Frunzio, L., Huang, R.-S., Majer, J., Kumar, S., Girvin, S.M., Schoelkopf, R.J.: Nature (London) **431**, 162 (2004)
21. Blais, A., Huang, R.-S., Wallraff, A., Girvin, S.M., Schoelkopf, R.J.: Phys. Rev. A **69**, 062320 (2004)
22. Wang, Y.D., Wang, Z.D., Sun, C.P.: Phys. Rev. B **72**, 172507 (2005)
23. Blais, A., Gambetta, J., Wallraff, A., Schuster, D.I., Girvin, S.M., Devoret, M.H., Schoelkopf, R.J.: Phys. Rev. A **75**, 032329 (2007)
24. Zhu, S.-L., Wang, Z.D., Yang, K.: Phys. Rev. A **68**, 034303 (2003)
25. Xue, Z.-Y., Wang, Z.D., Zhu, S.-L.: Phys. Rev. A **77**, 024301 (2008)
26. Xue, Z.-Y., Zhu, S.-L., You, J.Q., Wang, Z.D.: Phys. Rev. A **79**, 040303(R) (2009)
27. Xue, Z.-Y., Zhu, S.-L., Wang, Z.D.: Eur. Phys. J. D **55**, 223 (2009)
28. Zhu, S.-L., Wang, Z.D., Zanardi, P.: Phys. Rev. Lett. **94**, 100502 (2005)
29. Xue, Z.-Y., Wang, Z.D.: Phys. Rev. A **75**, 064303 (2007)
30. Liu, Y.-X., Sun, C.P., Nori, F.: Phys. Rev. A **74**, 052321 (2006)
31. Liu, Y.-X., Wei, L.F., Nori, F.: Phys. Rev. A **71**, 063820 (2005)
32. Liu, Y.-X., Wei, L.F., Johansson, J.R., Tsai, J.S., Nori, F.: Phys. Rev. B **76**, 144518 (2007)
33. Wei, L.F., Liu, Y.-X., Nori, F.: Phys. Rev. B **71**, 134506 (2005)
34. Zhu, S.-L., Zanardi, P.: Phys. Rev. A **72**, 020301(R) (2005)
35. Chen, G., Chen, Z., Yu, L., Liang, J.: Phys. Rev. A **76**, 024301 (2007)